

Rossmoyne SHS
Mathematics
Department

MATHEMATICS SPECIALIST 3A

Semester 1
2011
EXAMINATION

NAME: SOLUTIONS

TEACHER (circle): Mr White Ms Rigelsford
Ms Belonogoff Ms Robinson Mr Jones

Section One: Calculator-free

Time allowed for this section

Reading time before commencing work: 5 minutes

Working time for this section: 50 minutes

Material required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, eraser, correction fluid, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be used in this section of the examination. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available
Section One: Calculator-free	7	7	50	40
Section Two: Calculator-assumed	11	11	100	80
				120

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2011*. Sitting this examination implies that you agree to abide by these rules.
2. Answer the questions in the spaces provided.

Section One: Write answers in this Question/Answer Booklet. Answer **all** questions.

Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

It is recommended that you **do not use pencil**, except in diagrams.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

QUESTION	MARKS AVAILABLE	STUDENT MARK
1	7	
2	3	
3	3	
4	9	
5	8	
6	6	
7	4	
TOTAL	40	

The following exact value table may be useful to answer questions in this examination.

	0°	30°	45°	60°	90°
Sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Question 1

[3, 2 & 2 = 7 marks]

i) Use an algebraic method to solve for exact values of x .

a) $|3x - 2| = |x - 4|$

$$\begin{aligned} x < \frac{2}{3} \\ -(3x - 2) &= -(x - 4) \\ 3x - 2 &= x - 4 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

$$\begin{aligned} \frac{2}{3} \leq x < 4 \\ 3x - 2 &= -(x - 4) \\ 3x - 2 &= -x + 4 \\ 4x &= 6 \\ x &= \frac{3}{2} \end{aligned}$$

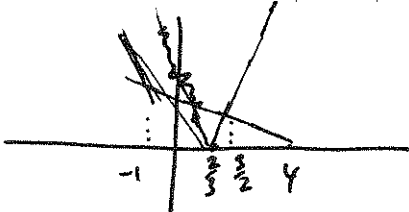
$$\begin{aligned} x \geq 4 \\ 3x - 2 &= x - 4 \\ 2x &= -2 \\ x &= -1 \\ \text{discard.} \end{aligned}$$

OR $3x - 2 = \pm(x - 4)$
 $x = -1, \frac{3}{2}$
 ✓ 2 marks for working
 ✓ 1 mark for answer

b) $|3x - 2| \geq |x - 4|$

$$x \leq -1, x \geq \frac{3}{2}$$

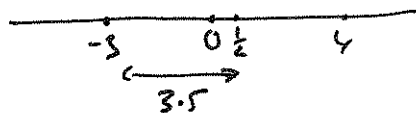
✓ answer only



ii) Determine the value of the constant c in the inequality

$|2x - 1| \leq c$ for which the solution set is $-3 \leq x \leq 4$

$$\left|x - \frac{1}{2}\right| \leq \frac{c}{2}$$



$$\frac{c}{2} = 3.5$$

$c = 7$ answer only ✓✓

Question 2

[3 marks]

Simplify the following.

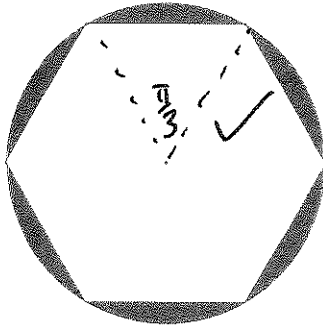
$$\begin{aligned} \frac{\sin \frac{\pi}{3} - \cos \frac{\pi}{3}}{\cos \frac{\pi}{6} + \sin \frac{\pi}{6}} &= \frac{\frac{\sqrt{3}}{2} - \frac{1}{2}}{\frac{\sqrt{3}}{2} + \frac{1}{2}} = \frac{(\sqrt{3} - 1) \cdot \frac{1}{2}}{(\sqrt{3} + 1) \cdot \frac{1}{2}} \quad \checkmark \\ &= \frac{3 - 2\sqrt{3} + 1}{3 - 1} \\ &= \frac{4 - 2\sqrt{3}}{2} \quad \checkmark \\ &= 2 - \sqrt{3} \quad \checkmark \end{aligned}$$

Question 3

[3 marks]

The diagram below shows a circular entertainment area. It has a paved hexagonal area with plants growing in the garden (shown as the shaded areas). If the radius of the circle is 6 m show that the area of the garden is

$$(36\pi - 54\sqrt{3})m^2.$$



$$\begin{aligned} \text{segment} &= \frac{1}{2} R^2 [\theta - \sin\theta] \\ &= \frac{1}{2} 6^2 \left[\frac{\pi}{3} - \sin \frac{\pi}{3} \right] \checkmark \\ \text{Total Area} &= 18 \left[\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right] \times 6 \\ &= 36\pi - 54\sqrt{3} \checkmark \end{aligned}$$

Question 4

[2, 2, 1, 2 & 2 = 9 marks]

i) Simplify each of the following.

$$\text{a) } \frac{5p^7q^3}{9r} \div \frac{(p^2q)^3}{-2r} = \frac{5p^7q^3}{9r} \cdot \frac{-2r}{p^6q^3} = \frac{-10p}{9} \checkmark$$

$$\text{b) } \frac{3^{x+2} + 45}{2(3^x) + 10} = \frac{9(3^x + 5)}{2(3^x + 5)} = \frac{9}{2} \checkmark$$

ii) Let $a = \log 2$ and $b = \log 5$. Express each of the following in terms of a and b and in the absence of log terms.

$$\text{a) } \log 8 + \log 25 = 3\log 2 + 2\log 5 = 3a + 2b \checkmark$$

$$\text{b) } 3 = 3\log_{10} 10 = 3(\log 2 + \log 5) = 3(a + b) \checkmark$$

$$\begin{aligned} \text{c) } \log_{16} 0.2 = x & \quad 16^x = 0.2 \checkmark \\ x = \frac{\log 0.2}{\log 16} &= \frac{\log \frac{1}{5}}{\log 2^4} = \frac{-\log 5}{4\log 2} = \frac{-b}{4a} \checkmark \end{aligned}$$

Question 5

[2, 3 & 3 = 8 marks]

Let vector $\vec{S} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and vector $\vec{T} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. Determine the following:

a) $|\vec{S} + 2\vec{T}| = \left| \begin{pmatrix} 1 \\ 8 \end{pmatrix} \right| = \sqrt{1^2 + 8^2} = \sqrt{65} \checkmark$

b) a vector parallel to \vec{S} but 5 times the length of \vec{T} .

$5\sqrt{65} \frac{1}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \sqrt{65} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \checkmark$

c) the values of λ and μ given that $\lambda\vec{S} + \mu\vec{T} + 2\lambda\vec{T} = 5\vec{S} + 4\vec{T} + \mu\vec{S}$

$(\lambda - 5 - \mu)\vec{S} = \vec{T}(4 - \mu - 2\lambda) \checkmark$

$\lambda - 5 - \mu = 0$

$4 - \mu - 2\lambda = 0$

$\lambda - \mu = 5 \dots \textcircled{1}$

$4 = \mu + 2\lambda$

$2\lambda + \mu = 4 \dots \textcircled{2}$

$3\lambda = 9$

$\textcircled{1} + \textcircled{2}$

$\lambda = 3 \checkmark$

$\mu = -2 \checkmark$

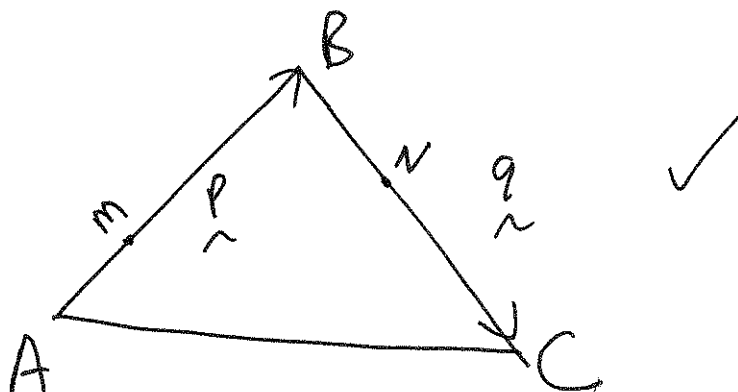
-1 for no working

Question 6

[1, 1, 2 & 2 = 6 marks]

ABC is a triangle with $\overline{AB} = \vec{p}$ and $\overline{BC} = \vec{q}$. In addition M divides AB in the ratio 1:3 and N divides BC in the ratio 2:3.

a) Draw a diagram that represents the information given.



Q6 Continued

b) Find in terms of \mathbf{p} and \mathbf{q} vector expressions for :

i) $\vec{AC} = \mathbf{p} + \mathbf{q}$ ✓

ii) $\vec{AN} = \mathbf{p} + \frac{2}{5}\mathbf{q}$ ✓✓

iii) $\vec{MC} = \frac{3}{4}\mathbf{p} + \mathbf{q}$ ✓✓

Question 7

[4 marks]

The function $f(x) = x^2 + 4x + 1$ has the domain $x \geq -2$ and the range $y \geq -3$.

Determine the inverse function $f^{-1}(x)$ stating its domain and range. (Simplified)

$$x = y^2 + 4y + 1$$

$$y^2 + 4y + 1 - x = 0 \quad \checkmark$$

$$y = \frac{-4 \pm \sqrt{16 - 4(1-x)}}{2}$$

$$= \frac{-4 \pm \sqrt{12 + 4x}}{2}$$

$$= \frac{-4 \pm 2\sqrt{3+x}}{2}$$

$$= -2 \pm \sqrt{3+x} \quad \checkmark$$

$$f^{-1}(x) = -2 + \sqrt{3+x} \quad \checkmark \quad \text{as domain: } x \geq -3 \quad \checkmark$$

$$\text{Range: } y \geq -2$$

End of Section 1

(-1 if not simplified)
 (-1 if used $-\sqrt{\quad}$)

Working out space
Please indicate on question if this space is used.

End of booklet